Let $\cos\beta\sin\theta=\sin\varphi$; then the second term becomes

$$2a^3\int_0^{\frac{1}{2}\pi-\beta}\sin^4\varphi\cos^2\varphi\log\left[\frac{\cos\varphi\cos\beta+\sqrt{(\cos^2\beta-\sin^2\varphi)}}{\sin\beta\sin\varphi}\right]d\varphi=2a^3\int_0^{\frac{1}{2}\pi-\beta}(\tfrac{1}{16}\varphi-$$

$$\tfrac{1}{16}\sin\varphi\cos\varphi-\tfrac{1}{24}\sin^3\varphi\cos\varphi+\tfrac{1}{6}\sin^5\varphi\cos\varphi)\frac{\cos\beta d\varphi}{\sin\varphi\sqrt{(\cos^2\beta-\sin^2\varphi)}}.$$

$$\therefore\ V=\tfrac{1}{16}\pi a^3\cos^3\beta\cot^2\beta(1+\sin^2\beta)+\tfrac{1}{8}a^3\cos\beta\int_0^{\frac{1}{2}\pi}d\theta+\tfrac{1}{12}\cos^3\beta\int_0^{\frac{1}{2}\pi}\sin^2\theta d\theta$$

$$-\tfrac{1}{3}a^3\cos^5\beta\int_0^{\frac{1}{2}\pi}\sin^4\theta d\theta-\tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}.$$

$$\therefore\ V=\tfrac{1}{16}\pi a^3\cot\beta\operatorname{cosec}\beta-\tfrac{1}{24}\pi a^3\cos^3\beta$$

$$-\tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}.\quad \tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}$$

$$=\tfrac{1}{8}a^3\cos\beta\int_0^{\frac{1}{2}\pi}(1+\tfrac{2}{3}\cos^2\beta\sin^2\theta+\tfrac{8}{15}\cos^4\beta\sin^4\theta+\tfrac{16}{35}\cos^6\beta\sin^6\theta+\ldots.)d\theta$$

$$=\tfrac{1}{16}\pi a^3(\cos\beta+\tfrac{1}{3}\cos^3\beta+\tfrac{1}{5}\cos^5\beta+\tfrac{1}{7}\cos^7\beta+\ldots.)=\tfrac{1}{16}\pi a^3\log\cot\tfrac{1}{2}\beta.$$

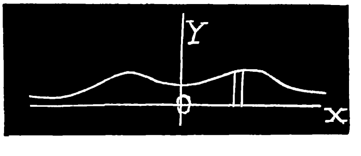
$$\therefore\ V=\tfrac{1}{48}\pi a^3(3\cot\beta\operatorname{cosec}\beta-2\cos^3\beta-3\log\cot\tfrac{1}{2}\beta).$$

150. Proposed by E. B. ESCOTT, Instructor in Mathematics, University of Michigan, Ann Arbor, Mich.

Find total area between the curve $x^4y-x^2+4y-1=0$ and the x-axis.

Solution by J. E. SANDERS, Hackney, Ohio, and the PROPOSER.

The equation may be written $y=\dfrac{x^2+1}{x^4+4}$.



$$\text{Area}=2\int_0^\infty ydx=2\int_0^\infty\frac{x^2+1}{x^4+4}dx$$

$$=\tfrac{1}{4}\left[\int_0^\infty\frac{x+2}{x^2-2x+2}dx-\int_0^\infty\frac{x-2}{x^2+2x+2}dx\right]=\tfrac{1}{8}\Big[\log(x^2-2x+2)$$

$$-\log(x^2+2x+2)+6[\tan^{-1}(x+1)+\tan^{-1}(x-1)]\Big]_0^\infty$$

$$=\tfrac{1}{8}\left[\log\frac{x^2-2x+2}{x^2+2x+2}+6\tan^{-1}\frac{2x}{2-x^2}\right]_0^\infty=\tfrac{1}{8}(6\pi)=\tfrac{3}{4}\pi.\quad \text{Answer.}$$

$\left[\tan^{-1}\frac{2x}{2-x^2}\right]_{x=\infty}=\tan^{-1}0=\pi$, since the integrand has increased and passed through ∞ for $x=\sqrt{2}$.

Also solved by *G. B. M. ZERR.*

MECHANICS.

144. Proposed by G. B. M. ZERR, A. M., Ph. D , Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Pressure is applied perpendicularly to the plane surface yz, bounding an otherwise infinite isotropic solid. Find the resultant displacements, if the pressure varies as $\sin\left(\frac{2\pi y}{a}\right)+\sinh\left(\frac{2\pi y}{a}\right)$.

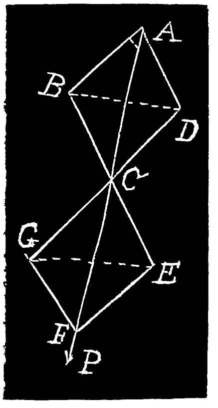
No solution of this problem has been received.

145. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

$ABCD$, $GCEF$ are equal parallelograms, DCG and BCE being straight lines. If the figure be considered as formed of smooth light jointed bars and if BD be a light rod, and the whole be suspended from A, find the stress in BD if a weight be hung from F. Also find the stress if a light rod GE replace BD.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics. The Temple College, Philadelphia, Pa.

Since the bars are light we can disregard their weight. Let P be the weight. Then by virtual work



$$Pd(AC)+Sd(BD)=0\ldots.(1).$$

But $AC^2+BD^2=2AD^2+2DC^2$.

$\therefore ACd(AC)+BDd(BD)=0\ldots.(2)$.

From (1) and (2),

$$\frac{d(AC)}{d(BD)}=-\frac{S}{P}=-\frac{BD}{AC}.$$

$\therefore S=\dfrac{P.BD}{AC}$. Similarly for GE.

$S_1=\dfrac{P.GE}{CF}$. The stress is the same for both.

146. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A diffraction grating, with lines .05 mm. apart is held in front of a Bunsen's burner in which common salt is volatilized, and, when viewed through a telescope it is found that the angular distances of the first, second, third, fourth, fifth, and sixth bright bands from the central one are respectively 41′, 1°21′, 2°2′, 2°42′, 3°23′ and 4°3′. Required the wave length of sodium light.